Fraction Division

Question:

How do you know that the answer to 5 ÷ 1/3 = 15?

Possible answers to the question are:

1. That doesn't make sense. You can't get a bigger answer when dividing because you are making a number into small pieces.
2. If you cut a pie into one-third size pieces, you will get 3 such pieces. Five pies will give you 5 x 3 = 15 pieces.
3. I remember a teacher telling me to invert and multiply. But what do you invert? I guess it must be the 1/3 because inverting the 5 is ugly. Inverting 1/3 and multiplying gives you 3/1 x 5 = 15.

Response 1 is all too typical. There are usually two sources of the misconception:

- Some early years teacher told the student that division gives you a smaller answer.
- The meaning of division is fixated on "sharing cookies equally " kind of thinking. This traps the student into whole number thinking for which answers must be smaller.

Response 2 provides an explanation for why the answer of 15 makes sense. It is based on the part of a whole meaning of fraction.

Response 3 is a procedural response. It offers no explanation for why the answer makes sense. Also, it reveals possible "bug" thinking that can occur when teachers tell students how to do arithmetic where the procedures do not make sense to students.

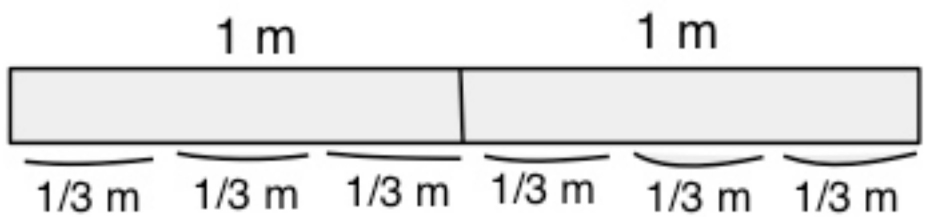
Models for teaching fraction division

A variety of teaching models can be used to develop fraction multiplication. The measure and part of a whole meanings of fraction can be useful when the division involves a whole number divided by a fraction. (see: Five meanings of fraction). The part of a set/group meaning of fraction is not appropriate for developing fraction division. It leads to a confusing situation that does not model division by fractions in an easily understood manner.

Fraction bar (or circles) model (for whole ÷ fraction)

This model involves the part of a whole meaning of fraction. An example for 2 ÷ 1/3 involving fraction bars follows.

Suppose you have a board 2 metres long. You cut the board into equal pieces, each of which is 1/3 metre long. How many 1/3 m size pieces can you get? This question is about splitting up into equal parts, and thus it concerns division. The appropriate number sentence is 2 ÷ 1/3 = ?. To obtain an answer to the division, the model would involve making as many 1/3 metre size pieces from the 2 metre board as possible and then counting up how many pieces are made. The diagram shows 6 pieces are made. Thus, 2 ÷ 1/3 = 6.



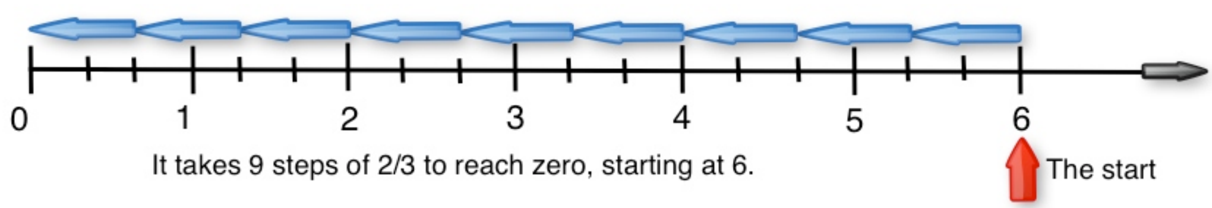
Number line model (for whole ÷ fraction)

This model involves the measure meaning of fraction. For multiplication, the number line model involves starting at zero and taking the number of steps (groups) indicated by the whole number multiplier. The answer is where you end up.

Because division is the reverse operation to multiplication, you reverse the process on the number line when dividing. You start at the whole number and take steps of size indicated by the fraction to reach zero. The number of steps taken is the answer to the division. There can be a remainder.

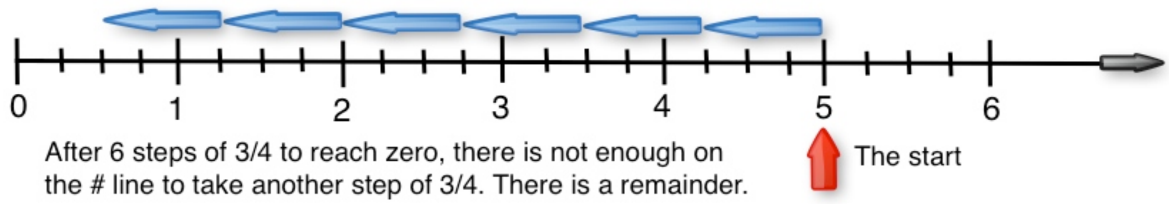
Two examples follow:

- The arithmetic works out nicely. Here is an example for 6 ÷ 2/3.



Starting at 6, it takes 9 steps of 2/3 to reach zero. Thus, 6 ÷ 2/3 = 9.

- The arithmetic does not work out nicely. The question is 5 ÷ 3/4



Starting at 5, it takes 5 steps of 3/4 to get as close to zero as possible. There is a remainder. What is the complete answer though?

Let's use the standard algorithm for dividing fractions (invert and multiply) to obtain an answer to the division.

5 ÷ 3/4 is the same as 5 x 4/3. This works out to be 20/3 or 6 2/3.

Note that 2/3 is not a remainder. 6 2/3 is the number of steps of size 3/4 needed to reach zero. Multiply 6 2/3 x 3/4. Do you get 5 as an answer?

Back to the remainder. The number line shows a remainder of 2/4 (or 1/2)

Thus, another way to express the answer for 5 ÷ 3/4 is 6 with remainder 2/4.

6 steps of 3/4 add the reminder of 2/4 should work out to equal 5. Does it?

Conclusion?

There are two ways to express the answer when there is a remainder.

The answer 6 2/3 (obtained by using the fraction division algorithm) tells you the number of steps of 3/4. The answer 6 with remainder 2/4 (obtained by using the number line) tells you how many whole steps and partial whole steps needed to reach zero.

The 2/3 part of 6 2/3 and the remainder 2/4 can be shown to be the same thing. But that adventure is for another day. But you could try doing 2/4 ÷ 3/4 and see what you get as an answer.

Refer to: Grade 8 Fraction division (8.N.6) **if more help is needed.**